Reg. No. | | | | | | | | | | | | |

Question Paper Code : X 60766

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2020
Second Semester
Civil Engineering
MA 2161/080030004/MA 22 – MATHEMATICS – II
(Common to all Branches)
(Regulations 2008)

Time : Three Hours | Maximum : 100 Marks

Answer ALL questions.

PART – A **(10×2=20 Marks)**

1. Solve $(D^2 - 4)y = 1$.

2. Convert into $(3x^2D^2 + 5xD + 7)y = 2/x\log x$ an equation with constant coefficients.

3. Find the directional derivative of $\phi = xyz$ at $(1,1,1)$ in the direction of $\vec{i} + \vec{j} + \vec{k}$.

4. If $\vec{A}$ and $\vec{B}$ are irrotational, prove that $\vec{A} \times \vec{B}$ is solenoidal.

5. State the basic difference between the limit of a function of a real variable and that of a complex variable.

6. Prove that a bilinear transformation has at most two fixed points.

7. What is meant by essential singularity ? Give an example.

8. State Cauchy's residue theorem.

9. What is meant by exponentially ordered function ?

10. Evaluate $\int_0^\infty \frac{1 - e^{-t}}{t}$ dt by using Laplace Transform.

PART – B **(5×16=80 Marks)**

11. a) i) Solve $(D^2 + a^2)y = \sec ax$ using the method of variation of parameters. **(8)**

ii) Solve : $(D^2 - 4D + 3)y = e^x \cos 2x$. **(8)**

(OR)

b) i) Solve the differential equation $(x^2D^2 - xD + 4)y = x^2 \sin(\log x)$. **(8)**

ii) Solve the simultaneous differential equations

$\frac{dx}{dt} + 2y = \sin 2t, \frac{dy}{dt} - 2x = \cos 2t.$ **(8)**

12. a) i) Show that the vector field $\bar{F} = (x^2 + xy^2)\bar{i} + (y^2 + x^2y)\bar{j}$ is irrotational. Find its scalar potential. **(6)**

ii) Verify Stoke's theorem for $\bar{F} = (x^2 + y^2)\bar{i} - 2xy\bar{j}$ taken around the rectangle formed by the lines $x = -a, x = +a, y = 0$ and $y = b$ **(10)**

(OR)

b) i) Find a and b so that the surface $ax^3 - by^2z - (a + 3)x^2 = 0$ and $4x^2y - z^3 - 11 = 0$ cut orthogonally at the point (2, –1, –3). **(6)**

ii) Verify Gauss Divergence theorem for $\bar{F} = 4xz\bar{i} - y^2\bar{j} + yz\bar{k}$, where S is the surface of the cube formed by the planes x = 0, x = 1, y = 0, y = 1 ; z = 0 and z =1. **(10)**

13. a) i) If u(x, y) and v(x, y) are harmonic functions in a region R, prove that

$\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) + i\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$ is analytic. **(8)**

ii) Find the bilinear transformation which transform the points $z = 0, 1, \infty$ into $w = i, -1, -i$ respectively. **(8)**

(OR)

b) i) If f(z) = u + iv is analytic, find f(z) given that $u + v = \frac{\sin 2x}{\cosh 2y - \cos 2x}$. **(8)**

ii) Under the transformation $w = \frac{1}{z}$ determine the region in w– plane of the infinite strip bounded by $\frac{1}{4} \le y \le \frac{1}{2}$. **(8)**

14. a) i) Use Cauchy's integral formula to evaluate $\int_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-2)(z-3)} dz$, where C is the circle $|z|=4$. **(8)**

ii) Find the Laurent's expansion of $f(z)=\frac{7z-2}{(z+1)z(z-2)}$ in the region $1<|z+1|<3$. **(8)**

(OR)

b) i) Using Cauchy's residue theorem, to evaluate $\int_c \frac{z-3}{z^2+2z+5} dz$. where C is the circle $|z|=3$. **(8)**

ii) Evaluate $\int_0^{2\pi} \frac{\sin^2 \theta}{5-3\cos\theta} d\theta$, by using contour integration technique. **(8)**

15. a) i) Find $L\left[t^2 e^{-3t} \sin 2t\right]$. **(8)**

ii) Find the Laplace transform of the square-wave function (or Meoander function) of period a defined as

$$f(t)=\begin{cases} 1, & \text{when } 0<t<\frac{a}{2} \\ -1, & \text{when } \frac{a}{t}<t<a \end{cases}.$$ **(8)**

(OR)

b) i) Using convolution theorem find the inverse Laplace transform of

$$\frac{4}{(s^2+2s+5)^2}.$$ **(8)**

ii) Solve $y''+5y'+6y=2$ given $y'(0)=0$ and $y(0)=0$ using Laplace transform. **(8)**